EXPERIMENTAL VALIDATION OF NONLINEAR DIPOLAR METHOD

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1. ABSTRACT

The non-linear dipolar method is a modelling method to accurately simulate quartz crystal oscillators. The main idea is to look upon the amplifier as a non-linear dipole. This is achieved by replacing the resonator by a current source and by performing a series of transient analyses of larger and larger amplitudes at the crystal resonant frequency. The method has been successfully implemented in the ADOQ software. It enables the user to compute the steady state features of the oscillator, namely the oscillation frequency and amplitude as well as the start-up condition for oscillation. In addition, the program performs accurate oscillator sensitivity calculation to various parameters (component value, supply voltage…). In this paper, we present an experimental validation of the simulation results provided by ADOQ software [2]. A measurement has been performed to obtain the equivalent non-linear impedance of the amplifier. The comparison between simulation and experimental results shows a very good agreement.

2. INTRODUCTION

Because of the very high circuit Q-factor, the simulation of quartz crystal oscillator fails with classical non-linear time domain simulation method while the dipolar method is well suited to the modelling of very high Q quartz crystal oscillators. This method allows to analyse the oscillator behaviour in relatively moderate calculation time. The principle has been widely used in the past [4, 5] and the method presents a renewed interest due to the present computer capability. Section 3 shows the details of this method applied to the quartz crystal behaviour analysis. Both steady state and start–up features are obtained using the non-linear dipolar method [1].

In section 4, we will show the capability of ADOQ software [2] to help the designer in checking or improving oscillator circuit design. Several examples of component influence to the oscillator operation are given in this section.

The measurement results presented in section 5 indents to assess the accuracy or simply validity limits of the dipolar method on the quartz crystal oscillator design.

3. NON-LINEAR DIPOLAR METHOD

The non-linear dipolar method consists in representing an oscillator by the quartz motional branch connected across a non-linear dipole amplifier (Fig. 1). The parallel capacitance and, if needed, the pulling capacitance are included in the amplifier dipole. The impedance of the non-linear dipole amplifier strongly depends on the current amplitude and weakly depends on the current frequency. It can be represented by a non-linear resistance in series with a non-linear inductance that vary with the amplitude of the current.

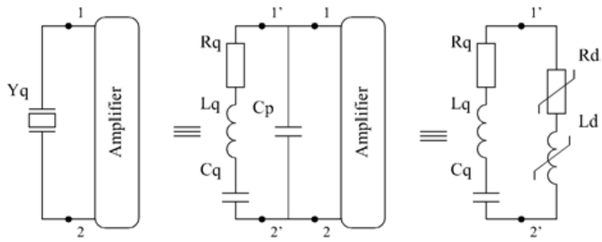

Fig. 1

Because of the high resonator's quality factor, the loop current in the oscillator is almost perfectly sinusoidal. Thus, the non-linear behaviour of the dipolar amplifier can be obtained by replacing the resonator motional branch by a sinusoidal current source, and performing a set of transient analyses with increasing amplitudes by using an electrical simulator like SPICE (Fig. 2).

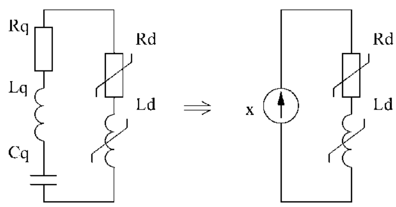

Fig. 2

The complex impedance of the non-linear dipole amplifier is obtained, for each current amplitude value by performing a Fourier analysis on the steady–state voltage across the dipole. Non-linear amplifier resistance and reactance are obtained by giving the sinusoidal current a larger and larger amplitude (Fig. 3).

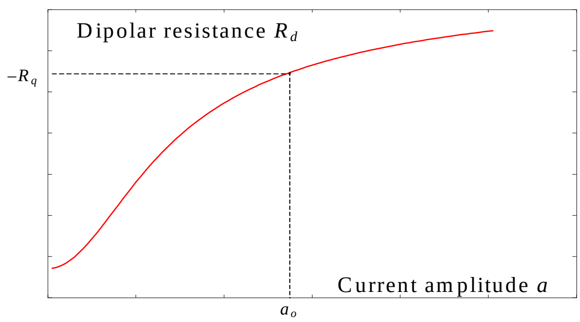

Fig. 3.a

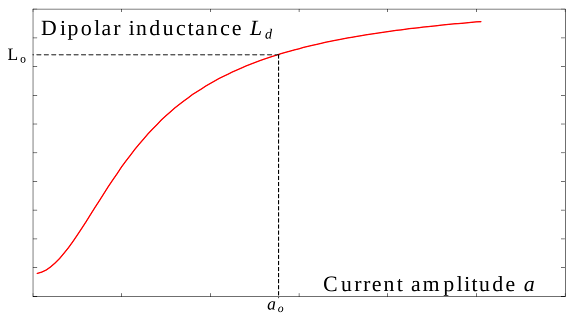

Fig. 3.b

The non-linear differential equation of the oscillator is given by Eq. (1), where ω_q is the quartz series resonant frequency and a is the amplitude of the current fundamental component $x(t)$. In quartz crystal oscillator $L_q >> L_d$ so that:

$$\frac{d^2x}{dt^2}+\frac{R_q+R_d(a)}{L_q}\frac{dx}{dt}+\omega_q^2\left(1-\frac{L_d(a)}{L_q}\right)x=0 \quad (1)$$

$$\omega_q^2=\frac{1}{L_qC_q} \quad (2)$$

The solution of this equation is taken under the form shown in Eq. (3) where $a(t)$ and $\phi(t)$ are slowly varying functions of time.

$$x(t)=a(t)\cos(\omega_q t+\phi(t)) \quad (3)$$

At low current amplitude level, the damping term of Eq. (1) should have a negative value to insure increasing amplitude solution. If R_{ds} is the value of the non-linear dipolar resistance at very low current amplitude, the start-up condition takes the form shown in Eq. (4).

$$R_q+R_{ds}<0 \quad (4)$$

As the oscillation amplitude increases, the dipolar resistance increases so that the value of the damping term increases. The steady state amplitude a_o is reached when this term becomes zero (Eq. 5 and Fig. 3.a).

$$R_q+R_d(a_o)=0 \quad (5)$$

$$\omega_o^2=\omega_q^2\left(1-\frac{L_d(a_o)}{L_q}\right) \quad (6)$$

The steady state frequency is then given by Eq. 6 where $L_d(a_o)$ is the value of the steady state dipolar inductance deduced from the curve in Fig. 3.b.

4. SIMULATION RESULTS

As shown in section 3, the non-linear dipolar method, allows us to determine the start-up condition as well as the steady state frequency and amplitude using popular electrical simulator like SPICE in a moderate calculation time. However, it is more interesting to use this method to optimise the oscillator circuit design [2]. The first step of the optimisation process is to evaluate the sensitivity of the oscillator operating condition as a function of oscillator component variation.

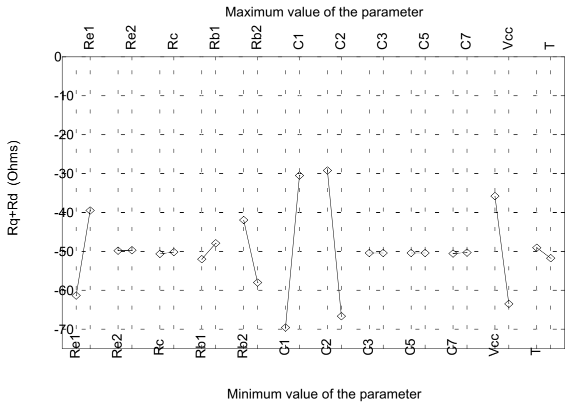

Fig. 4

Fig. 4 shows the loop resistance variation in start-up condition for a shift of ±10% of the oscillator component values. The loop resistance value should remain negative (Eq. 4) to insure the start-up of the oscillation. This parametric analysis allows us to choose the circuit components so that oscillation starts in all condition.

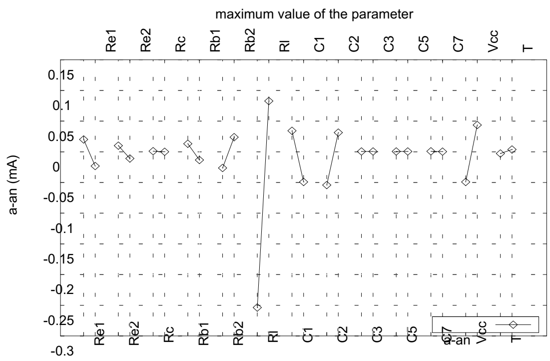

Fig. 5.a

The steady state is reached when the real part of the loop impedance is zero (Eq. 5). The oscillation amplitude and frequency sensitivities are obtained by varying several oscillator components. The results are shown in Fig. 5a and Fig. 5b.

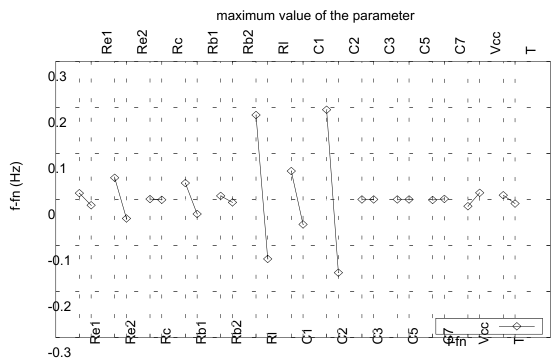

Fig. 5.b

The amplitude and frequency shifts are shown for a variation of ±10% of each parameter around their nominal value. The results that looks like a parametric analysis of the amplitude and frequency oscillation allows us to identify the most sensitive component that should be carefully chosen.

5. EXPERIMENTAL VALIDATION

The experimental setup is based on a *Agilent Technologies* impedance analyser (4395A). It can perform harmonic impedance measurements of a dipolar device as a function of the analysis power. The impedance analyser is connected to a PC computer via a GPIB (General Purpose Interface Bus) bus. A software program was developed to control impedance analyser. The frequency and other analysis parameters, as well as the operating instructions, are monitored via this program. In the case of this particular impedance analyser design, the power range is limited between -50dBm and 15dBm. In practice, the dipolar impedance is measured by using 5 steps of 13dBm power range span. The calibration of the analyser is needed for both power range and frequency. All calibration data are stored in the computer in order to restore them for the same intrinsic operating conditions during different measurements.

The precision of the dipolar method is strongly depending on the modelling accuracy of the amplifier. Before comparing the simulation results with the experimental data, it is necessary to evaluate the inherent modelling uncertainty which is due to the dispersion of the components used as well as of the printed circuit board parasitic elements.

The choice of the MAT03 transistor used is motivated because of the very low dispersion characteristics and the availability of an accurate SPICE model. On the one hand, the very low dispersion of this transistor family minimises the dissonance between the simulation and the experimental results when the transistor is changed, on the other hand, the availability of the SPICE model guaranties an optimal accuracy in the modelling of the oscillator amplifier. The oscillator used is a 10MHz to 20MHz Colpitts oscillator proposed in [3].

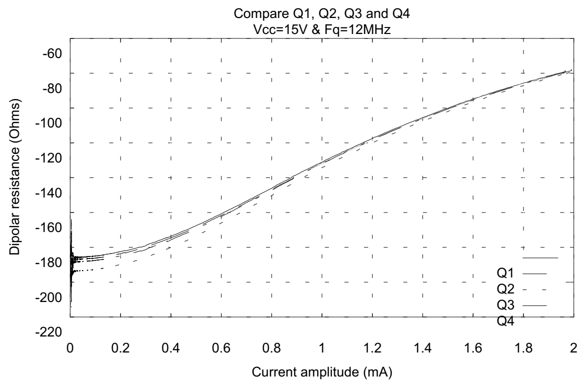

Fig. 6

First, several dipolar impedance measurements were performed using different transistors (Fig. 6). It is shown that the spreading that is larger than 5% for low level amplitude dipolar resistance reduces to less than 1% around the steady-state.

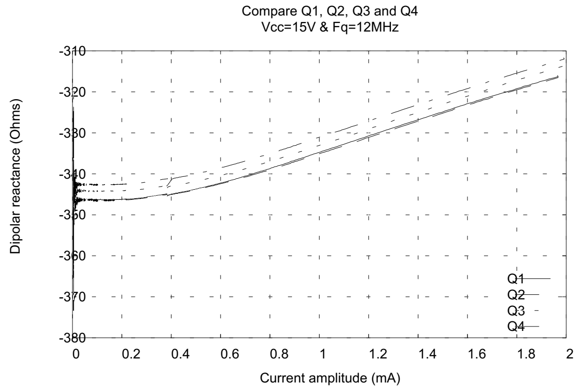

Fig. 7

The dipolar reactance spreading is close to 1% over the whole analysis current amplitude range (Fig. 7). These measurements give an order of magnitude of the discrepancies that could be expected between simulation and experimental results.

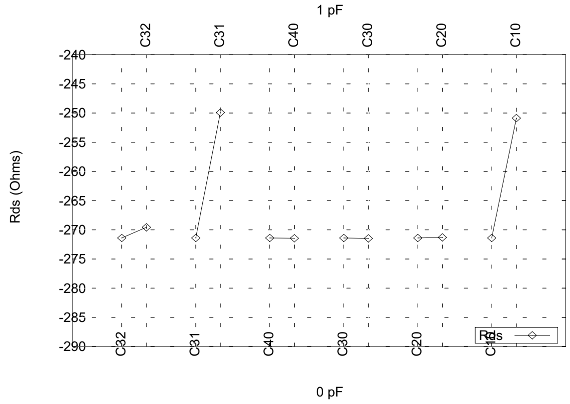

Fig. 8.a

The oscillator circuit is described in SPICE file format where the component values are given the measured ones. The influence of parasitic capacitances on the real and imaginary part of dipolar impedance in start-up condition is demonstrated in Fig. 8a and Fig. 8b where several intertrack capacitance have been varied between 0 and 1pF.

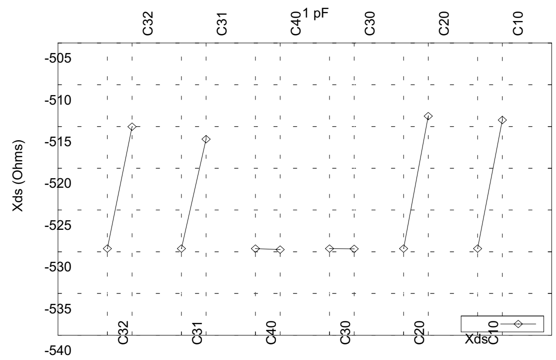

Fig. 8.b

The results shows that the shift due to those parasitic capacitances should not to be neglected. The parasitic capacitances are then measured and included in the file describing the circuit.

```
-----------------------------------------2
-2222018642. D2i 18p42oDl ar. .  2D erl l s. Dt 2222-2
-----------------------------------------2
nee2   c2     12     ( 0c2
2
t 02   02     12     01O2
t i 2  02     c2     0hmi O2
t ) 2  C2     c2     umdhO2
t l 2  ) 2    12     c12
2
l 02   C2     i 2    0A62
2
e02    02     i 2    ) i Qa, 2
ei 2   i 2    ) 2    01m) a, 2
e) 2   ) 2    12     0cca, 2
eC2    i 2    ) 2    00mQa, 2
22
-2ast s  r. r e2esaser. s3eV 2
e012   02     12     ca2
ei 12  i 2    12     Qmca2
e) 12  ) 2    12     umca2
eC12   C2     12     umca2
e) 02  ) 2    02     0a2
e) i 2 ) 2    i 2    ) mQa2
2
```

Fig. 9

The supply voltage source used is monitored by the PC. Several dipolar impedance measurements are then performed for different values of the supply voltage. The results are shown in Fig. 10 for a Colpitts oscillator operating at 12MHz and ambient temperature of 25°C. It can be observed that the difference between simulation and measurements remains in the expected limits.

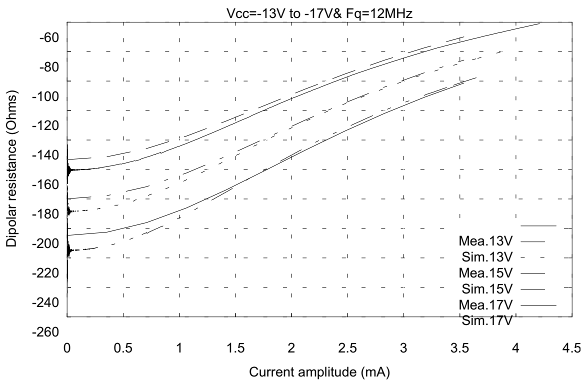

Fig. 10

In the same way measurements have been performed by varying the analysis frequency. The calculation of the oscillation frequency needs several simulations in the neighbouring of the intrinsic resonator frequency.

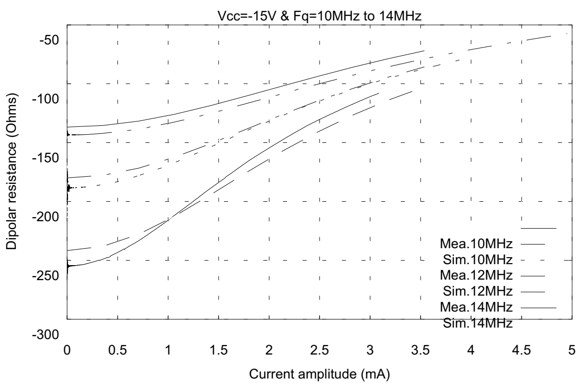

Fig. 11

The measurements shown in (Fig. 11) are performed for several analysis frequencies. It seems that the results are less good than the previous measurements. The spreading is close to 3% at low level amplitude measurement and remains below 1.5% at higher amplitude level. It is even so heartening results.

The Colpitts oscillator built is mounted in temperature controlled oven. An example of measurements of the dipolar impedance obtained for two temperatures is shown in Fig. 12. The shift that is about 5% at low level amplitude reduces to less then 1% at higher values. When performing the simulation, the curves seems rotate around –75 Ohms point. This rotation is shown also on the measured dipolar resistance.

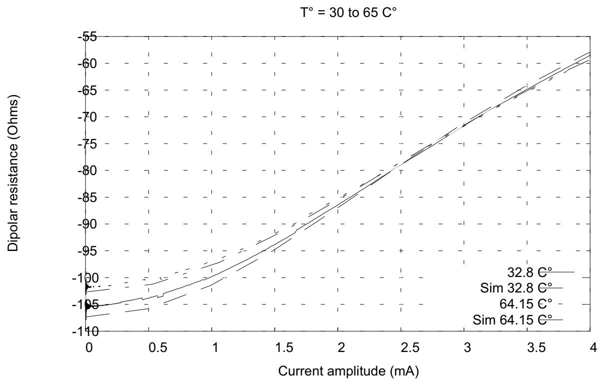

Fig. 12

For the three monitored parameters, simulation and measurement results show a pretty good agreement.

6. CONCLUSION

Although it still exist small discrepancies between simulation and experimental results, the agreement is good enough to validate the approach used in the non-linear dipolar method, and demonstrates the reliability of the sensitivity analysis results.

The accuracy of the oscillator features obtained from the simulation can be sufficient when the inherent dissonances are widely assumed and the circuit modelling is precisely performed.

8. REFERENCES

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